1.2.5 Exercises

In Exercises 1 - 6, graph the function. Find the slope and axis intercepts, if any.

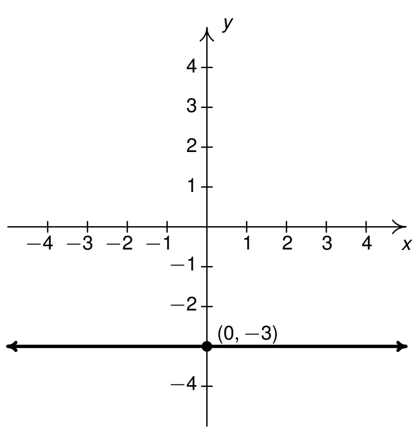
1. $f(x) = 2x - 1$
2. $g(t) = 3 - t$
3. $F(w) = 3$
4. $G(s) = 0$
5. $h(t) = \frac{2}{3}t + \frac{1}{3}$
6. $j(w) = \dfrac{1-w}{2}$

In Exercises 7 - 10, graph the function. Find the domain, range, and axis intercepts, if any.

7. $f(x) = \begin{cases} 4 - x & \text{if } x \leq 3 \\ 2 & \text{if } x > 3 \end{cases}$
8. $g(x) = \begin{cases} 2 - x & \text{if } x < 2 \\ x - 2 & \text{if } x \geq 2 \end{cases}$
9. $F(t) = \begin{cases} -2t - 4 & \text{if } t < 0 \\ 3t & \text{if } t \geq 0 \end{cases}$
10. $G(t) = \begin{cases} -3 & \text{if } t < 0 \\ 2t - 3 & \text{if } 0 < t < 3 \\ 3 & \text{if } t > 3 \end{cases}$
11. The **unit step function** is defined as $U(t) = \begin{cases} 0 & \text{if } t < 0, \\ 1 & \text{if } t \geq 0. \end{cases}$
    (a) Graph $y = U(t)$.
    (b) State the domain and range of U.
    (c) List the interval(s) over which U is increasing, decreasing, and/or constant.
    (d) Write $U(t-2)$ as a piecewise defined function and graph.

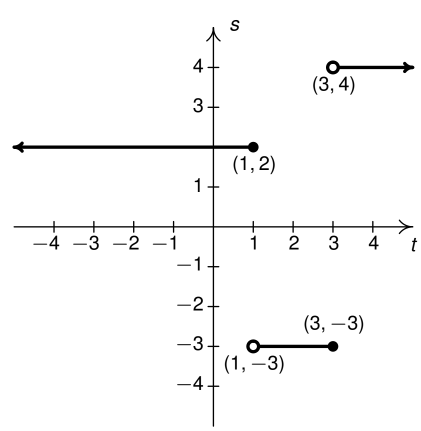
In Exercises 12 - 15, find a formula for the function.

12.



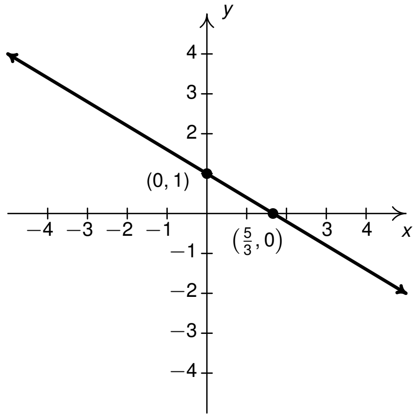
$y = f(x)$

13.



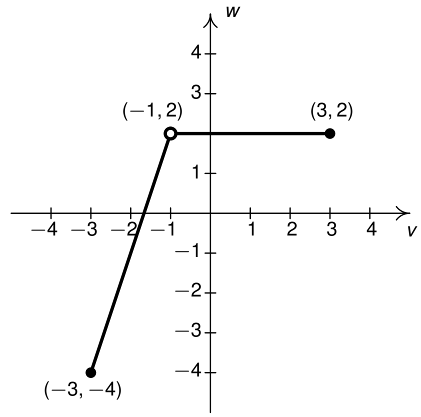
$s = F(t)$

14.



$y = L(x)$

15.



$w = g(v)$

16. For n copies of the book *Me and my Sasquatch*, a print on-demand company charges $C(n)$ dollars, where $C(n)$ is determined by the formula

$$C(n) = \begin{cases} 15n & \text{if} \quad 1 \leq n \leq 25 \\ 13.50n & \text{if} \quad 25 < n \leq 50 \\ 12n & \text{if} \quad n > 50 \end{cases}$$

(a) Find and interpret $C(20)$.

(b) How much does it cost to order 50 copies of the book? What about 51 copies?

(c) Your answer to 16b should get you thinking. Suppose a bookstore estimates it will sell 50 copies of the book. How many books can, in fact, be ordered for the same price as those 50 copies? (Round your answer to a whole number of books.)

17. An on-line comic book retailer charges shipping costs according to the following formula

$$S(n) = \begin{cases} 1.5n + 2.5 & \text{if} \quad 1 \leq n \leq 14 \\ 0 & \text{if} \quad n \geq 15 \end{cases}$$

where n is the number of comic books purchased and $S(n)$ is the shipping cost in dollars.

(a) What is the cost to ship 10 comic books?

(b) What is the significance of the formula $S(n) = 0$ for $n \geq 15$?

18. The cost in dollars $C(m)$ to talk m minutes a month on a mobile phone plan is modeled by

$$C(m) = \begin{cases} 25 & \text{if} \quad 0 \leq m \leq 1000 \\ 25 + 0.1(m - 1000) & \text{if} \quad m > 1000 \end{cases}$$

   (a) How much does it cost to talk 750 minutes per month with this plan?
   (b) How much does it cost to talk 20 hours a month with this plan?
   (c) Explain the terms of the plan verbally.

19. Jeff can walk comfortably at 3 miles per hour. Find an expression for a linear function $d(t)$ that represents the total distance Jeff can walk in t hours, assuming he doesn't take any breaks.

20. Carl can stuff 6 envelopes per *minute*. Find an expression for a linear function $E(t)$ that represents the total number of envelopes Carl can stuff after t *hours*, assuming he doesn't take any breaks.

21. A landscaping company charges \$45 per cubic yard of mulch plus a delivery charge of \$20. Find an expression for a linear function $C(x)$ which computes the total cost in dollars to deliver x cubic yards of mulch.

22. A plumber charges \$50 for a service call plus \$80 per hour. If she spends no longer than 8 hours a day at any one site, find an expression for a linear function $C(t)$ that computes her total daily charges in dollars as a function of the amount of time spent in hours, t at any one given location.

23. A salesperson is paid \$200 per week plus 5% commission on her weekly sales of x dollars. Find an expression for a linear function $W(x)$ which computes her total weekly pay in dollars as a function of x. What must her weekly sales be in order for her to earn \$475.00 for the week?

24. An on-demand publisher charges \$22.50 to print a 600 page book and \$15.50 to print a 400 page book. Find an expression for a linear function which models the cost of a book in dollars $C(p)$ as a function of the number of pages p. Find and interpret both the slope of the linear function and $C(0)$.

25. The Topology Taxi Company charges \$2.50 for the first fifth of a mile and \$0.45 for each additional fifth of a mile. Find an expression for a linear function which models the taxi fare $F(m)$ as a function of the number of miles driven, m. Find and interpret both the slope of the linear function and $F(0)$.

26. Water freezes at 0° Celsius and 32° Fahrenheit and it boils at 100°C and 212°F.

   (a) Find an expression for a linear function $F(T)$ that computes temperature in the Fahrenheit scale as a function of the temperature T given in degrees Celsius. Use this function to convert 20°C into Fahrenheit.
   (b) Find an expression for a linear function $C(T)$ that computes temperature in the Celsius scale as a function of the temperature T given in degrees Fahrenheit. Use this function to convert 110°F into Celsius.
   (c) Is there a temperature T such that $F(T) = C(T)$?

27. Legend has it that a bull Sasquatch in rut will howl approximately 9 times per hour when it is $40^\circ F$ outside and only 5 times per hour if it's $70^\circ F$. Assuming that the number of howls per hour, N, can be represented by a linear function of temperature Fahrenheit, find the number of howls per hour he'll make when it's only $20^\circ F$ outside. What troubles do you encounter when trying to determine a reasonable applied domain?

28. Economic forces have changed the cost function for PortaBoys to $C(x) = 105x + 175$. Rework Example 1.2.3 with this new cost function.

29. In response to the economic forces in Exercise 28 above, the local retailer sets the selling price of a PortaBoy at \$250. Remarkably, 30 units were sold each week. When the systems went on sale for \$220, 40 units per week were sold. Rework Example 1.2.4 with this new data.

30. A local pizza store offers medium two-topping pizzas delivered for \$6.00 per pizza plus a \$1.50 delivery charge per order. On weekends, the store runs a 'game day' special: if six or more medium two-topping pizzas are ordered, they are \$5.50 each with no delivery charge. Write a piecewise-defined linear function which calculates the cost in dollars $C(p)$ of p medium two-topping pizzas delivered during a weekend.

31. A restaurant offers a buffet which costs \$15 per person. For parties of 10 or more people, a group discount applies, and the cost is \$12.50 per person. Write a piecewise-defined linear function which calculates the total bill $T(n)$ of a party of n people who all choose the buffet.

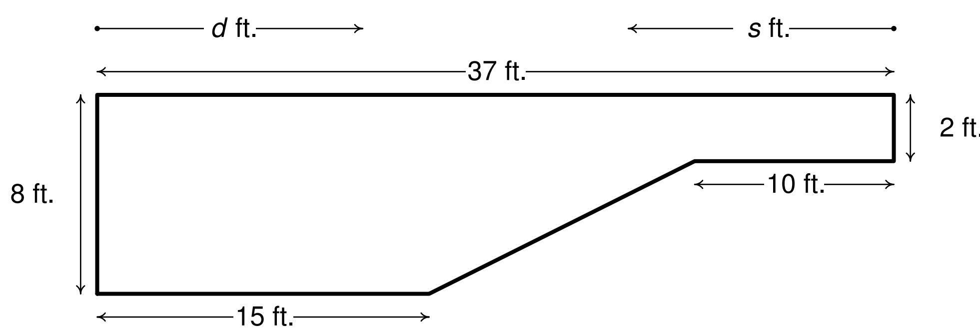
32. A mobile plan charges a base monthly rate of \$10 for the first 500 minutes of air time plus a charge of 15¢ for each additional minute. Write a piecewise-defined linear function which calculates the monthly cost in dollars $C(m)$ for using m minutes of air time.

    HINT: You may wish to refer to number 18 for inspiration.

33. The local pet shop charges 12¢ per cricket up to 100 crickets, and 10¢ per cricket thereafter. Write a piecewise-defined linear function which calculates the price in dollars $P(c)$ of purchasing c crickets.

34. The cross-section of a swimming pool is below. Write a piecewise-defined linear function which describes the depth of the pool, D (in feet) as a function of:

    (a) the distance (in feet) from the edge of the shallow end of the pool, d.
    (b) the distance (in feet) from the edge of the deep end of the pool, s.
    (c) Graph each of the functions in (a) and (b). Discuss with your classmates how to transform one into the other and how they relate to the diagram of the pool.



35. The function defined by $I(x) = x$ is called the Identity Function. Thinking from a procedural perspective, explain a possible origin of this name.

36. Why must the graph of a function $y = f(x)$ have at most one y-intercept?

    HINT: Consider what would happen graphically if there were more than one . . .

37. Why is a discussion of vertical lines omitted when discussing functions?

38. Find a formula for the x-intercept of the graph of $f(x) = mx + b$. Assume $m \neq 0$.

39. Suppose $(c, 0)$ is the x-intercept of a linear function f. Use the point-slope form of a liner function, Equation 1.1 to show $f(x) = m(x - c)$. This is the 'slope x-intercept' form of the linear function.

40. Prove that for all linear functions L with with slope 3, $L(120) = L(100) + 60$.

41. Find the slopes between the following points from the data set given in Example 1.2.7 and compare them with the slope of the corresponding regression line:

    (a) $(0, 64)$, $(4, 75)$ (b) $(4, 75)$, $(8, 83)$ (c) $(8, 83)$, $(10, 83)$ (d) $(10, 83)$, $(12, 82)$

42. According to this website[31], the census data for Lake County, Ohio is:

Year	1970	1980	1990	2000
Population	197200	212801	215499	227511

    (a) Find the least squares regression line for these data and comment on the goodness of fit.[32] Interpret the slope of the line of best fit.
    (b) Use the regression line to predict the population of Lake County in 2010. (The recorded figure from the 2010 census is 230,041)
    (c) Use the regression line to predict when the population of Lake County will reach 250,000.

43. According to this website[33], the census data for Lorain County, Ohio is:

Year	1970	1980	1990	2000
Population	256843	274909	271126	284664

    (a) Find the least squares regression line for these data and comment on the goodness of fit. Interpret the slope of the line of best fit.
    (b) Use the regression line to predict the population of Lorain County in 2010. (The recorded figure from the 2010 census is 301,356)

[31] http://www.ohiobiz.com/census/Lake.pdf

[32] We'll develop more sophisticated models for the growth of populations in Chapter 6. For the moment, we use a theorem from Calculus to approximate those functions with lines.

[33] http://www.ohiobiz.com/census/Lorain.pdf

(c) Use the regression line to predict when the population of Lake County will reach 325,000.

44. The chart below contains a portion of the fuel consumption information for a 2002 Toyota Echo that Jeffrey used to own. The first row is the cumulative number of gallons of gasoline that I had used and the second row is the odometer reading when I refilled the gas tank. So, for example, the fourth entry is the point (28.25, 1051) which says that I had used a total of 28.25 gallons of gasoline when the odometer read 1051 miles.

Gasoline Used (Gallons)	0	9.26	19.03	28.25	36.45	44.64	53.57	62.62	71.93	81.69	90.43
Odometer (Miles)	41	356	731	1051	1347	1631	1966	2310	2670	3030	3371

Find the least squares line for this data. Is it a good fit? What does the slope of the line represent? Do you and your classmates believe this model would have held for ten years had I not crashed the car on the Turnpike a few years ago?

45. Using the energy production data given below

Year	1950	1960	1970	1980	1990	2000
Production (in Quads)	35.6	42.8	63.5	67.2	70.7	71.2

(a) Plot the data using a graphing utility and explain why it does not appear to be linear.

(b) Discuss with your classmates why ignoring the first two data points may be justified from a historical perspective.

(c) Find the least squares regression line for the last four data points and comment on the goodness of fit. Interpret the slope of the line of best fit.

(d) Use the regression line to predict the annual US energy production in the year 2010.

(e) Use the regression line to predict when the annual US energy production will reach 100 Quads.

In Exercises 46 - 51, compute the average rate of change of the function over the specified interval.

46. $f(x) = x^3,\ [-1, 2]$

47. $g(x) = \dfrac{1}{x},\ [1, 5]$

48. $f(t) = \sqrt{t},\ [0, 16]$

49. $g(t) = x^2,\ [-3, 3]$

50. $F(s) = \dfrac{s+4}{s-3},\ [5, 7]$

51. $G(s) = 3s^2 + 2s - 7,\ [-4, 2]$

52. The height of an object dropped from the roof of a building is modeled by: $h(t) = -16t^2 + 64$, for $0 \leq t \leq 2$. Here, $h(t)$ is the height of the object off the ground in feet t seconds after the object is dropped. Find and interpret the average rate of change of h over the interval $[0, 2]$.

53. Using data from Bureau of Transportation Statistics, the average fuel economy $F(t)$ in miles per gallon for passenger cars in the US can be modeled by $F(t) = -0.0076t^2 + 0.45t + 16$, $0 \leq t \leq 28$, where t is the number of years since 1980. Find and interpret the average rate of change of F over the interval $[0, 28]$.

54. The temperature $T(t)$ in degrees Fahrenheit t hours after 6 AM is given by:

$$T(t) = -\frac{1}{2}t^2 + 8t + 32, \quad 0 \leq t \leq 12$$

   (a) Find and interpret $T(4)$, $T(8)$ and $T(12)$.
   (b) Find and interpret the average rate of change of T over the interval $[4, 8]$.
   (c) Find and interpret the average rate of change of T from $t = 8$ to $t = 12$.
   (d) Find and interpret the average rate of temperature change between 10 AM and 6 PM.

55. Suppose $C(x) = x^2 - 10x + 27$ represents the costs, in *hundreds*, to produce x *thousand* pens. Find and interpret the average rate of change as production is increased from making 3000 to 5000 pens.

56. Recall from Example 1.2.8 The formula $s(t) = -5t^2 + 100t$ for $0 \leq t \leq 20$ gives the height, $s(t)$, measured in feet, of a model rocket above the Moon's surface as a function of the time after lift-off, t, in seconds.

   (a) Find and interpret the average rate of change of s over the following intervals:

      i. $[14.9, 15]$ ii. $[15, 15.1]$ iii. $[14.99, 15]$ iv. $[15, 15.01]$

   (b) What value does the average rate of change appear to be approaching as the interval shrinks closer to the value $t = 15$?
   (c) Find the equation of the line containing $(15, 375)$ with slope $m = -50$ and graph it along with s on the same set of axes using a graphing utility. What happens as you zoom in near $(15, 375)$?

57. Show the average rate of change of a function of the form $f(x) = mx + b$ over *any* interval is m.

58. Why doesn't the graph of the vertical line $x = b$ in the xy-plane represent y as a function of x?

59. With help from a graphing utility, graph the following pairs of functions on the same set of axes:[34]

   - $f(x) = 2 - x$ and $g(x) = \lfloor 2 - x \rfloor$
   - $f(x) = x^2 - 4$ and $g(x) = \lfloor x^2 - 4 \rfloor$
   - $f(x) = x^3$ and $g(x) = \lfloor x^3 \rfloor$
   - $f(x) = \sqrt{x} - 4$ and $g(x) = \lfloor \sqrt{x} - 4 \rfloor$

   Choose more functions $f(x)$ and graph $y = f(x)$ alongside $y = \lfloor f(x) \rfloor$ until you can explain how, in general, one would obtain the graph of $y = \lfloor f(x) \rfloor$ given the graph of $y = f(x)$.

[34] See Example 1.2.2 for the definition of $\lfloor x \rfloor$.

60. The Lagrange Interpolate function L for two points (x_0, y_0) and (x_1, y_1) where $x_0 \neq x_1$ is given by:

$$L(x) = y_0 \frac{x - x_1}{x_0 - x_1} + y_1 \frac{x - x_0}{x_1 - x_0}$$

(a) For each of the following pairs of points, find $L(x)$ using the formula above and verify each of the points lies on the graph of $y = L(x)$.

i. $(-1, 3)$, $(2, 3)$ ii. $(-3, -2)$, $(5, -2)$ iii. $(-3, -2)$, $(0, 1)$ iv. $(-1, 5)$, $(2, -1)$

(b) Verify that, in general, $L(x_0) = y_0$ and $L(x_1) = y_1$.

(c) Show the point-slope form of a linear function, Equation 1.1 is equivalent to the formula given for $L(x)$ after making the identifications: $f(x_0) = y_0$ and $m = \dfrac{y_1 - y_0}{x_1 - x_0}$.