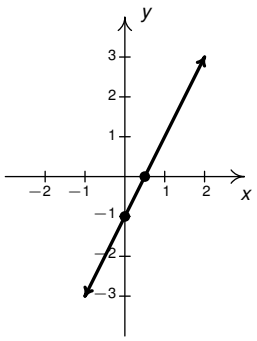
1.2.6 Answers

1. $f(x) = 2x - 1$

   slope: $m = 2$

   y-intercept: $(0, -1)$

   x-intercept: $\left(\frac{1}{2}, 0\right)$

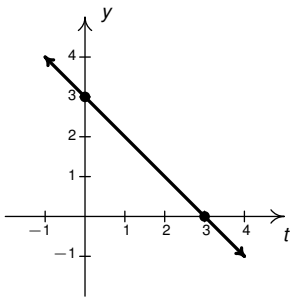


2. $g(t) = 3 - t$

   slope: $m = -1$

   y-intercept: $(0, 3)$

   t-intercept: $(3, 0)$

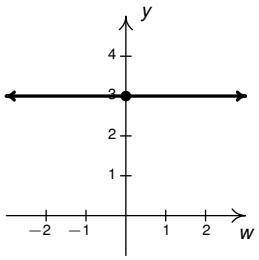


3. $F(w) = 3$

   slope: $m = 0$

   y-intercept: $(0, 3)$

   w-intercept: none

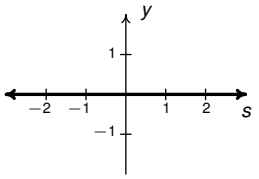


4. $G(s) = 0$

   slope: $m = 0$

   y-intercept: $(0, 0)$

   s-intercept: $\{(s, 0) \mid s \text{ is a real number}\}$

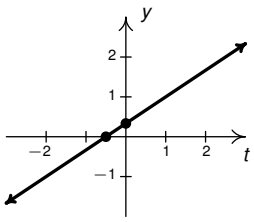


5. $h(t) = \frac{2}{3}x + \frac{1}{3}$

   slope: $m = \frac{2}{3}$

   y-intercept: $\left(0, \frac{1}{3}\right)$

   t-intercept: $\left(-\frac{1}{2}, 0\right)$

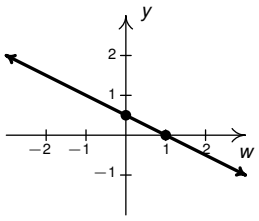


6. $j(w) = \dfrac{1 - w}{2}$

   slope: $m = -\frac{1}{2}$

   y-intercept: $\left(0, \frac{1}{2}\right)$

   w-intercept: $(1, 0)$



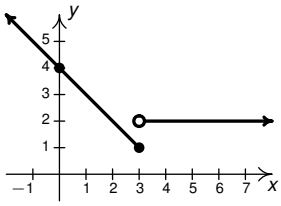
7.

   domain: $(-\infty, \infty)$

   range: $[1, \infty)$

   y-intercept: $(0, 4)$

   x-intercept: none



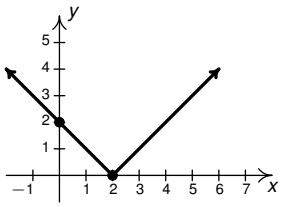
8.

   domain: $(-\infty, \infty)$

   range: $[0, \infty)$

   y-intercept: $(0, 2)$

   x-intercept: $(2, 0)$



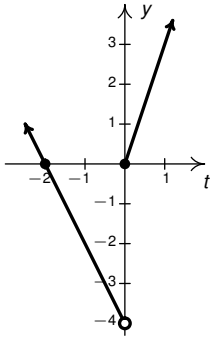
9.

   domain: $(-\infty, \infty)$

   range: $(-4, \infty)$

   y-intercept: $(0, 0)$

   t-intercepts: $(-2, 0)$, $(0, 0)$



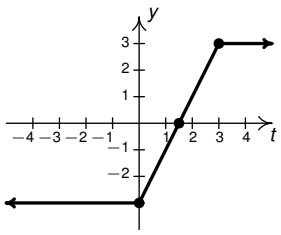
10.

domain: $(-\infty, \infty)$

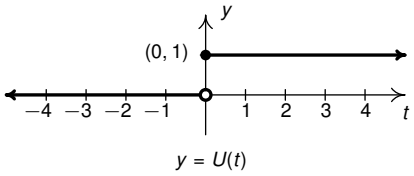
range: $[-3, 3]$

y-intercept: $(0, -3)$

t-intercept: $\left(\frac{3}{2}, 0\right) = (1.5, 0)$



11. (a)

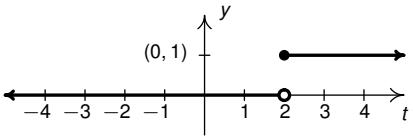


$y = U(t)$

(b) domain: $(-\infty, \infty)$, range: $\{0, 1\}$

(c) U is constant on $(-\infty, 0)$ and $[0, \infty)$.

(d) $U(t-2) = \begin{cases} 0 & \text{if } t < 2, \\ 1 & \text{if } t \geq 2. \end{cases}$



$y = U(t-2)$

12. $f(x) = -3$

13. $F(t) = \begin{cases} 2 & \text{if } t \leq 1, \\ -3 & \text{if } 1 < t \leq 3, \\ 4 & \text{if } t > 3. \end{cases}$

14. $L(x) = -\frac{3}{5}x + 1$

15. $g(v) = \begin{cases} 3v + 5 & \text{if } -3 \leq v < -1, \\ 2 & \text{if } -1 < v \leq 3, \end{cases}$

16. (a) $C(20) = 300$. It costs \$300 for 20 copies of the book.

(b) $C(50) = 675$, \$675. $C(51) = 612$, \$612.

(c) 56 books.

17. (a) $S(10) = 17.5$, \$17.50.

(b) There is free shipping on orders of 15 or more comic books.

18. (a) $C(750) = 25$, \$25.

(b) $C(1200) = 45$, \$45.

(c) It costs \$25 for up to 1000 minutes and 10 cents per minute for each minute over 1000 minutes.

19. $d(t) = 3t$, $t \geq 0$.

20. $E(t) = 360t$, $t \geq 0$.

21. $C(x) = 45x + 20$, $x \geq 0$.

22. $C(t) = 80t + 50$, $0 \leq t \leq 8$.

23. $W(x) = 200 + .05x$, $x \geq 0$ She must make \$5500 in weekly sales.

24. $C(p) = 0.035p + 1.5$ The slope 0.035 means it costs 3.5¢ per page. $C(0) = 1.5$ means there is a fixed, or start-up, cost of \$1.50 to make each book.

25. $F(m) = 2.25m + 2.05$ The slope 2.25 means it costs an additional \$2.25 for each mile beyond the first 0.2 miles. $F(0) = 2.05$, so according to the model, it would cost \$2.05 for a trip of 0 miles. Would this ever really happen? Depends on the driver and the passenger, we suppose.

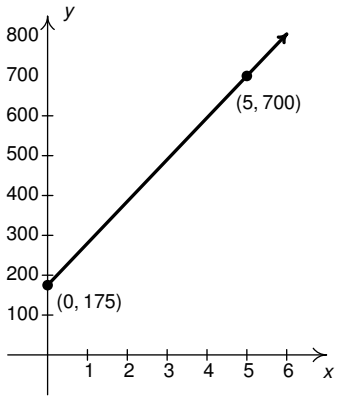
26. (a) $F(T) = \frac{9}{5}T + 32$ (b) $C(T) = \frac{5}{9}(T - 32) = \frac{5}{9}T - \frac{160}{9}$

    (c) $F(-40) = -40 = C(-40)$.

27. $N(T) = -\frac{2}{15}T + \frac{43}{3}$ and $N(20) = \frac{35}{3} \approx 12$ howls per hour.

    Having a negative number of howls makes no sense and since $N(107.5) = 0$ we can put an upper bound of $107.5^\circ F$ on the domain. The lower bound is trickier because there's nothing other than common sense to go on. As it gets colder, he howls more often. At some point it will either be so cold that he freezes to death or he's howling non-stop. So we're going to say that he can withstand temperatures no lower than $-42^\circ F$ so that the applied domain is $[-42, 107.5]$.

28. (a) $C(0) = 175$, so our start-up costs are \$175. $C(5) = 700$, so to produce 5 systems, it costs \$700.



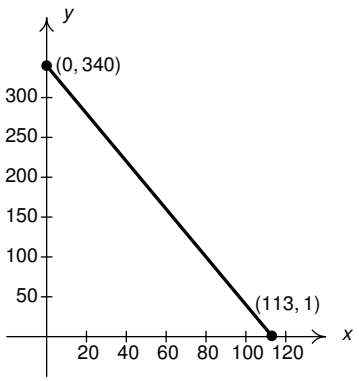
$y = C(x)$

    (b) Since we can't make a negative number of game systems, $x \geq 0$.

    (c) The slope is $m = 105$ so for each additional system produced, it costs an additional \$105.

    (d) Solving $C(x) = 15000$ gives $x \approx 141.19$ so 141 can be produced for \$15,000.

29. (a) $p(x) = -3x + 340$, $0 \le x \le 113$.



$y = p(x)$

(b) The slope is $m = -3$ so for each \$3 drop in price, we sell one additional game system.

(c) Since $x = 150$ is not in the domain of p, $p(150)$ is not defined. (In other words, under these conditions, it is impossible to sell 150 game systems.)

(d) Solving $p(x) = 150$ gives $x \approx 63.33$ so if the price \$150 per system, we would sell 63 systems.

30. $C(p) = \begin{cases} 6p + 1.5 & \text{if} \quad 1 \le p \le 5 \\ 5.5p & \text{if} \quad p \ge 6 \end{cases}$

31. $T(n) = \begin{cases} 15n & \text{if} \quad 1 \le n \le 9 \\ 12.5n & \text{if} \quad n \ge 10 \end{cases}$

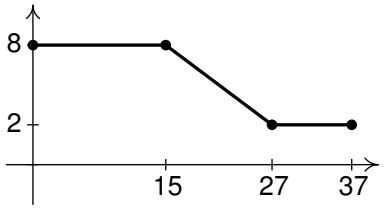
32. $C(m) = \begin{cases} 10 & \text{if} \quad 0 \le m \le 500 \\ 10 + 0.15(m - 500) & \text{if} \quad m > 500 \end{cases}$

33. $P(c) = \begin{cases} 0.12c & \text{if} \quad 1 \le c \le 100 \\ 12 + 0.1(c - 100) & \text{if} \quad c > 100 \end{cases}$

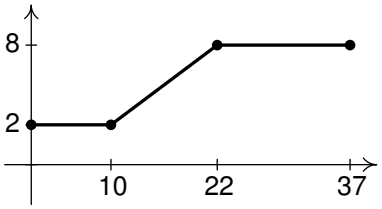
34. (a)
$$D(d) = \begin{cases} 8 & \text{if} \quad 0 \le d \le 15 \\ -\frac{1}{2}\,d + \frac{31}{2} & \text{if} \quad 15 \le d \le 27 \\ 2 & \text{if} \quad 27 \le d \le 37 \end{cases}$$

(b)
$$D(s) = \begin{cases} 2 & \text{if} \quad 0 \le s \le 10 \\ \frac{1}{2}\,s - 3 & \text{if} \quad 10 \le s \le 22 \\ 8 & \text{if} \quad 22 \le s \le 37 \end{cases}$$

(c)



$y = D(d)$



$y = D(s)$

35. Since $I(x) = x$ for all real numbers x, the function I doesn't change the 'identity' of the input at all.

36. If a graph contains more than one y-intercept, it would violate the Vertical Line Test since $x = 0$ would be matched with (at least) two different y-values.

37. Vertical Lines fail the Vertical Line Test.

38. $\left(-\frac{b}{m}, 0\right)$. (Note the importance here of $m \neq 0$.)

39. Plugging in $(c, 0)$ for $(x_0, f(x_0))$, we get $f(x) = f(x_0) + m(x - x_0) = 0 + m(x - c)$ or $f(x) = m(x - c)$.

40. Since L is linear with slope 3, $L(x) = L(x_0) + m\Delta x = L(100) + (3)(120 - 100) = L(100) + 60$.

41. (a) $m = \frac{75-64}{4-0} = 2.75$ (b) $m = \frac{83-75}{8-4} = 2$

    (c) $m = \frac{83-83}{10-8} = 0$ (d) $m = \frac{82-83}{12-10} = -0.5$

    The first two points contributed to a regression line slope of $m = 2.55$; the last two points contributed to a regression line slope of $m = -0.25$.

42. (a) $y = 936.31x - 1645322.6$ with $r = 0.9696$ which indicates a good fit. The slope 936.31 indicates Lake County's population is increasing at a rate of (approximately) 936 people per year.

    (b) According to the model, the population in 2010 will be 236,660.

    (c) According to the model, the population of Lake County will reach 250,000 sometime between 2024 and 2025.

43. (a) $y = 796.8x - 1309762.5$ with $r = 0.8916$ which indicates a reasonable fit. The slope 796.8 indicates Lorain County's population is increasing at a rate of (approximately) 797 people per year.

    (b) According to the model, the population in 2010 will be 291,805.

    (c) According to the model, the population of Lake County will reach 325,000 sometime between 2051 and 2052.

44. The regression line is $y = 36.8x + 16.39$ with $r = .99987$, so this is an excellent fit. The slope 36.8 represents mileage in miles per gallon.

45. (c) $y = 0.266x - 459.86$ with $r = 0.9607$ which indicates a good fit. The slope 0.266 indicates the country's energy production is increasing at a rate of 0.266 Quad per year.

    (d) According to the model, the production in 2010 will be 74.8 Quad.

    (e) According to the model, the production will reach 100 Quad in the year 2105.

46. $\frac{2^3-(-1)^3}{2-(-1)} = 3$

47. $\frac{\frac{1}{5}-\frac{1}{1}}{5-1} = -\frac{1}{5}$

48. $\frac{\sqrt{16}-\sqrt{0}}{16-0} = \frac{1}{4}$

49. $\frac{3^2-(-3)^2}{3-(-3)} = 0$

50. $\dfrac{\frac{7+4}{7-3} - \frac{5+4}{5-3}}{7-5} = -\frac{7}{8}$

51. $\dfrac{(3(2)^2+2(2)-7)-(3(-4)^2+2(-4)-7)}{2-(-4)} = -4$

52. The average rate of change is $\frac{h(2)-h(0)}{2-0} = -32$. During the first two seconds after it is dropped, the object has fallen at an average rate of 32 feet per second.

53. The average rate of change is $\frac{F(28)-F(0)}{28-0} = 0.2372$. From 1980 to 2008, the average fuel economy of passenger cars in the US increased, on average, at a rate of 0.2372 miles per gallon per year.

54. (a) $T(4) = 56$, so at 10 AM (4 hours after 6 AM), it is 56°F. $T(8) = 64$, so at 2 PM (8 hours after 6 AM), it is 64°F. $T(12) = 56$, so at 6 PM (12 hours after 6 AM), it is 56°F.

    (b) The average rate of change is $\frac{T(8)-T(4)}{8-4} = 2$. Between 10 AM and 2 PM, the temperature increases, on average, at a rate of 2°F per hour.

    (c) The average rate of change is $\frac{T(12)-T(8)}{12-8} = -2$. Between 2 PM and 6 PM, the temperature decreases, on average, at a rate of 2°F per hour.

    (d) The average rate of change is $\frac{T(12)-T(4)}{12-4} = 0$. Between 10 AM and 6 PM, the temperature, on average, remains constant.

55. The average rate of change is $\frac{C(5)-C(3)}{5-3} = -2$. As production is increased from 3000 to 5000 pens, the cost decreases at an average rate of \$200 per 1000 pens produced (20¢ per pen.)

56. (a) i. -49.5 so the average velocity of the rocket between 14.9 and 15 seconds after lift off is -49.5 feet per second (49.5 feet per second directed *downwards*.)

    ii. -50.5 so the average velocity of the rocket between 14 and 15.1 seconds after lift off is -50.5 feet per second. (50.5 feet per second directed *downwards*.)

    iii. -49.95 so the average velocity of the rocket between 14.99 and 15 seconds after lift off is -49.95 feet per second. (49.95 feet per second directed *downwards*.)

    iv. -50.05 so the average velocity of the rocket between 15.01 and 15 seconds after lift off is -50.05 feet per second. (50.05 feet per second directed *downwards*.)

    (b) The average rate of change seem to be approaching -50.

    (c) Line: $y = -50(t - 15) + 375$ or $y = -50t + 1125$. Graphing this line along with the s on a graphing utility we find the two graphs become indistinguishable as we zoom in near $(15, 375)$.

60. (a) i. $L(x) = 3$ ii. $L(x) = -2$ iii. $L(x) = x + 1$ iv. $L(x) = -2x + 3$