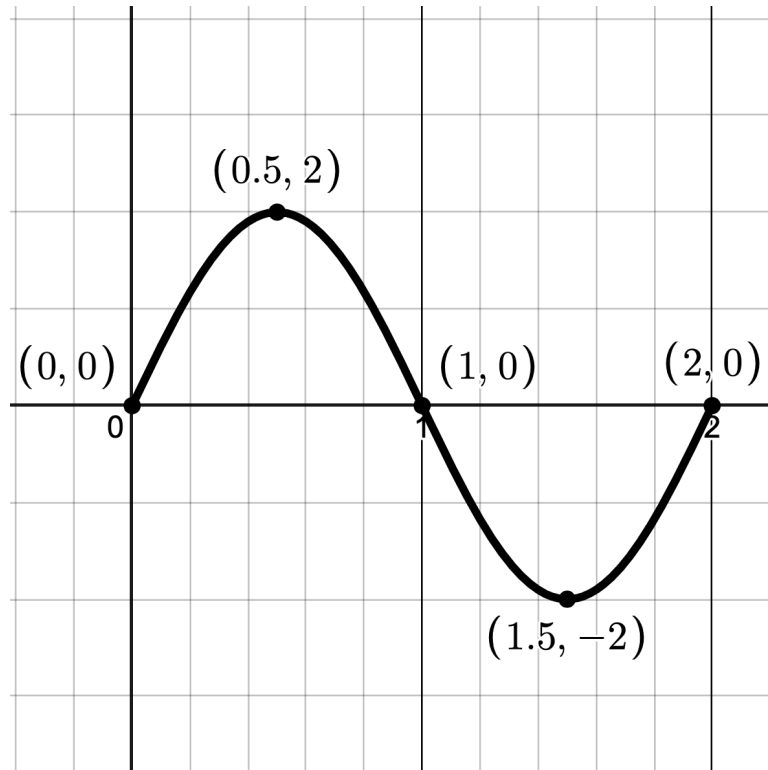


MATH 1650: TEST 03 (100 points.)

NAME: \_\_\_\_\_

**DIRECTIONS:** To receive full credit, make sure your work is neat and complete.

1. The complete graph of  $y = f(x)$  is below.



- (a) List the domain and range of  $f$  using interval notation.

• domain:

• range:

- (b) Find the maximum of  $f$ , if it exists:

Find the minimum of  $f$ , if it exists:

- (c) List the zeros of  $f$ :

- (d) List the intervals over which  $f$  is increasing and decreasing:

• increasing:

• decreasing:

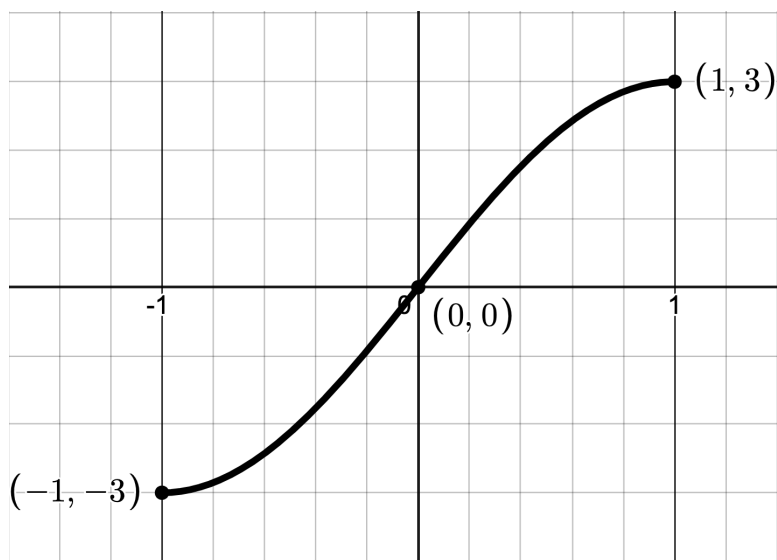
- (e) List the local maximums and minimums, if any.

• local maximum(s):

• local minimum(s):

- (f) Find  $(f \circ f)(0.5)$ .

2. The complete graph of  $y = f(x)$  is below.



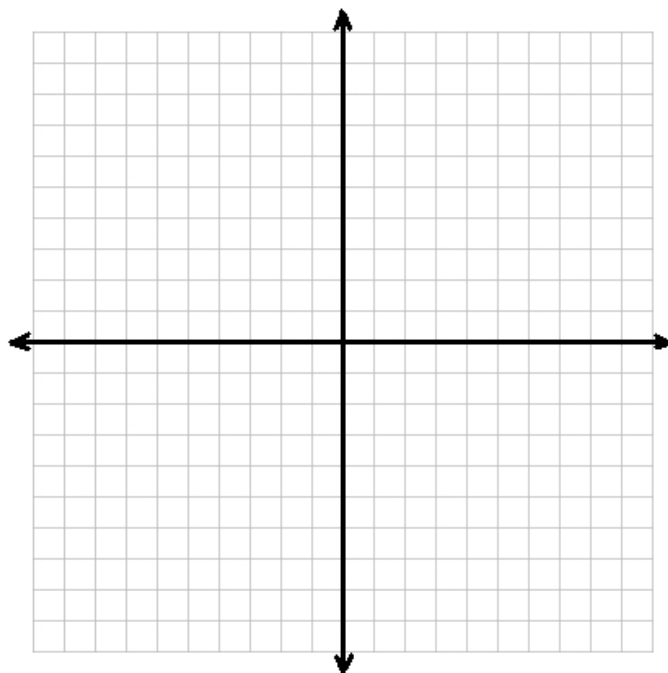
(a) List the domain and range of  $f$  using interval notation.

• domain:

• range:

(b) Does  $f$  appear to be even, odd, or neither?

(c) Explain why  $f$  appears to be invertible and graph  $y = f^{-1}(x)$  below. Label at least three points.



(d) List the domain and range of  $f^{-1}$  using interval notation.

• domain:

• range:

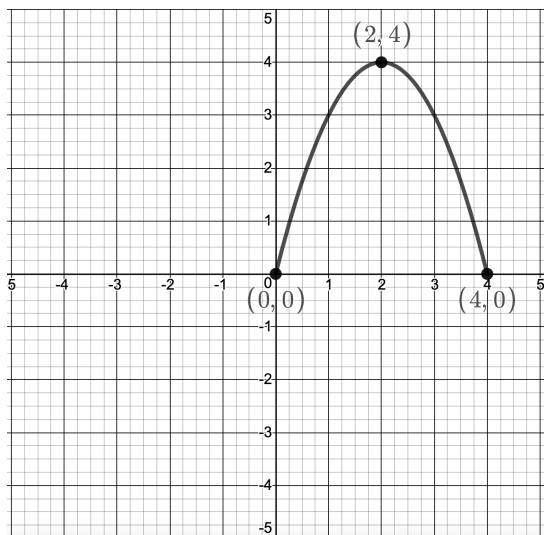
3. Let  $f(x) = x^2 - 4$  and  $g(x) = \sqrt{x + 4}$ . Find and simplify:

(a)  $(f \circ g)(x)$

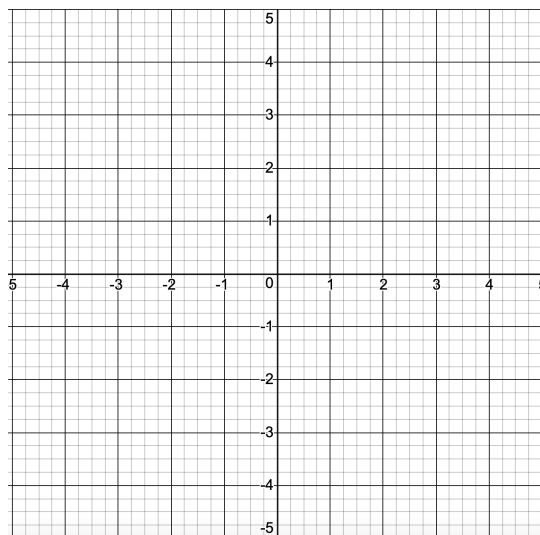
(b)  $(g \circ f)(x)$

Based on your answers, are  $f$  and  $g$  inverses? Explain.

4. Use the graph of  $f$  below on the left to graph the function below on the right. Label at least three points.



$$y = f(x)$$



$$y = 2f(x + 1) - 3$$

5. Let  $f(x) = e^{2x-1} + 3$

(a) Find a formula for  $f^{-1}(x)$ .

(b) Check your answer to part (a) algebraically by simplifying:

i.  $(f^{-1} \circ f)(x)$

ii.  $(f \circ f^{-1})(x)$

6. Write a sentence (or two!) to explain why ...

(a)  $\log_3(-2)$  isn't a real number.

(b)  $\log_3(x) = -\log_{\frac{1}{3}}(x)$

7. Fully expand and simplify using properties of logarithms:  $\log_2(16\sqrt{x})$

8. Write as a single logarithm:  $\log_3(x) - 2\log_3(y) - 1$

9. Use a Sign Diagram to help you find the domain of:  $f(x) = \ln \left( \frac{2x+1}{3-x} \right)$ .

10. Solve the equation:  $\log_{12}(2-x) + \log_{12}(3-x) = 1$ .

11. Let  $A(t) = 5000e^{0.015t}$ .

(a) Find and simplify:  $A(0)$ .

(b) i. Solve the equation  $A(t) = 10000$  algebraically. Find an exact answer and use your calculator to find a decimal approximation, rounded to two decimal places.

ii. Sketch the graph of  $y = A(t)$  and  $y = 10000$  and explain what your solution to part (i) means graphically.

(c) The function  $A(t)$  represents the amount of money (in dollars) in an account  $t$  years after it is invested. In this case, the interest is being compounded continuously.

i. What does the quantity  $A(0)$  represent in this situation?

ii. What does your solution to the equation  $A(t) = 10000$  mean in this situation?

12. The half-life of a radioactive compound is 4 days. Suppose 100 grams of the compound is available initially.

(a) How many grams are left after 12 days?

**HINT:** 12 days is how many half-lives?

(b) Find a function of the form  $A(t) = A_0 e^{-kt}$  where  $A(t)$  is the amount of the substance, in grams, after  $t$  days. Round  $k$  to three decimal places. Interpret  $k$  as a rate of change.