

MATH 2850: TAKE HOME 02 (25 points.)

NAME: _____

DUE: Wednesday, January 31st, at the beginning of class.

DIRECTIONS: Show all work.

1. Consider the IVP: $x \frac{dy}{dx} + y^2 + 1 = 0$, $y(1) = -1$.

Show this DE is separable and find an explicit solution to this IVP. Include an interval of validity.

2. Consider the DE: $xy^2 \frac{dy}{dx} = y^3 - 1$.

(a) Explain why this DE is not linear in the variable y .

(b) Show this DE is separable and find an explicit solution by separating variables.

Include an interval of validity.

3. Consider the DE: $xy^2 \frac{dy}{dx} = y^3 - 1$.

NOTE: This the same DE from #2.

(a) Explain why this DE is linear in the variable x .

(b) Solve this DE using an integrating factor.

BONUS: Show how you can rewrite your solution in #2 to match your solution here.

4. When forgetfulness is taken into account, the rate of memorization of a subject is modeled by:

$$\frac{dA}{dt} = k_1 (M - A) - k_2 A, \quad A(0) = 0,$$

where $A = A(t)$ is the amount memorized in time t , $M > 0$ is the total amount to be memorized (a constant), and k_1 and k_2 are two other positive constants.

(a) This equation is linear! Put the equation into standard form and find the integrating factor.

(b) Find the **explicit** formula for the general solution of the ODE.

(c) Impose the initial condition to find the **explicit** formula for the solution of the IVP.

(d) Find $\lim_{t \rightarrow \infty} A(t)$. Is it possible to memorize all of the material? Explain.

NOTE: Be sure to explain why it's important for your analysis that k_1 and k_2 are **positive**.