

MATH 2850: TAKE HOME 06 (30 points.)

NAME: _____

DUE: Wednesday, February 28th, at the beginning of class.

DIRECTIONS: Show all work.

1. Let's do some more work with substitutions:

(a) Suppose $x = e^t$ so that $t = \ln(x)$. Show $\frac{dy}{dx} = \frac{1}{x} \frac{dy}{dt}$.

HINT: Recall the Chain Rule says: $\frac{dy}{dx} = \frac{dy}{dt} \frac{dt}{dx} = \dots$

(b) Use the result in part (a) to show $\frac{d^2y}{dx^2} = \frac{1}{x^2} \frac{d^2y}{dt^2} - \frac{1}{x^2} \frac{dy}{dt}$

HINT: $\frac{d^2y}{dx^2} = \frac{d}{dx} \left[\frac{dy}{dx} \right] = \frac{d}{dx} \left[\frac{1}{x} \frac{dy}{dt} \right] = \dots$

Use the product rule then use part (a) to determine $\frac{d}{dx} \left[\frac{dy}{dt} \right]$.

(c) Make the substitutions you derived in parts (a) and (b) into the DE: $3x^2y'' + 4xy' - 2y = 0$ for $x > 0$.

Solve the resulting DE for $y = y(t)$ then substitute in for x to get the general solution, $y = y(x)$.

2. Consider an ODE of the form: $y' + y^2 + P(x)y + f(x) = 0$

(a) Prove the substitution $y = u'/u$ transforms the ODE into a homogeneous second order linear equation.

(b) Use the substitution in part (a) to solve: $y' + y^2 = y + 2$

NOTE: How many arbitrary constants should you have in your answer? What gives?