

SECTION 1.1

Part A: Sets of Numbers

SETS OF NUMBERS

- A set is a well-defined collection of objects which are called the 'elements' of the set. The order of the elements do not matter.
- 'Well-defined' means that it is possible to determine if something belongs to the collection or not.
- When an element belongs to a set, we use the symbol ' \in '
If the element x belongs to set A , mathematically $x \in A$.

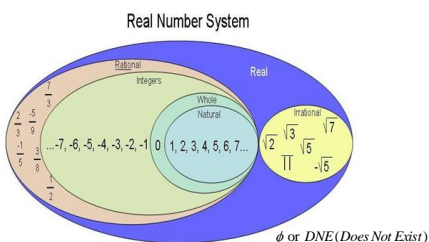
WAYS TO DESCRIBE SETS

- The Verbal Method: Use a sentence to define a set.
- The Roster Method: Begin with a left brace '{', list each element of the set only once and then end with a right brace '}'.
- The Set-Builder Method: A combination of the verbal and roster methods using a 'dummy variable' such as x .
- The Interval Notation Method: A method for writing an interval of numbers by using the endpoints of the interval.
- The Graphical Method: Graph interval of numbers on a number line.

SETS OF NUMBERS

- The Empty Set: $\emptyset = \{ \} = \{ x \mid x \neq x \}$. This is the set with no elements.
- The Natural Numbers: $\mathbb{N} = \{ 1, 2, 3, \dots \}$
- The Whole Numbers: $\mathbb{W} = \{ 0, 1, 2, 3, \dots \}$
- The Integers: $\mathbb{Z} = \{ \dots, -3, -2, -1, 0, 1, 2, 3, \dots \}$
- The Rational Numbers: $\mathbb{Q} = \{ m/n \text{ where } m, n \text{ are integers and } n \neq 0 \}$
- The Irrational Numbers: $\mathbb{P} = \{ \text{all numbers that cannot be written as a rational number} \}$
- The Real Numbers: $\mathbb{R} = \{ \text{all numbers on the number line; both rational and irrational} \}$.

REAL NUMBER SYSTEM



GRAPHICAL METHOD

In graphing a set of numbers on a number line, there are 5 things to draw:

- Draw a line segment in bold (or in color) to show an interval of numbers.
- Put an open circle at the end of a line segment to exclude the end point from the interval.
- Put a filled-in circle at the end of a line segment to include the end point of the interval.
- Put a left arrow at the end of a line segment to show that the segment continues to negative infinity.
- Put a right arrow at the end of a line segment to show that the segment continues to infinity.

SET-BUILDER NOTATION

In set-builder notation:

- Start with "{ x |" that means the set of x where conditions on x are to follow after the |. Note: x is a dummy variable and any dummy variable can be used.
- Conditions are expressed as equalities or inequalities of x , such as:
 - $x = 2$
 - $x < 2, x \leq 2, x > 2, x \geq 2$
 - $-2 < x \leq 3$
- Do not write inequalities using the ∞ symbol.
- Join equalities or inequalities with the word "or".
- Finish the notation with "}"

INTERVAL NOTATION

In interval notation, there are 5 symbols to know:

- Open parentheses excludes end points ()
- Closed brackets include end points []
- Infinity: ∞
- Negative Infinity: $-\infty$
- Union Symbol: \cup

INTERVAL NOTATION TIPS

- All numbers within the interval notation are in ascending numeric order from left to right.
- If the interval goes on forever to the left, start the interval notation with $(-\infty$
- Similarly, if the interval goes on forever to the right, end the interval notation with $\infty)$
- Whenever a graph consists of multiple line segments, write out each interval from left to right. Put the union \cup symbol between each interval to join them together.

INTERVAL NOTATION EXAMPLES

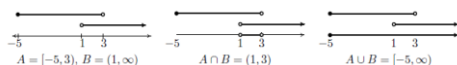
Interval Notation
Let a and b be real numbers with $a < b$.

Set of Real Numbers	Interval Notation	Region on the Real Number Line
$\{x a < x < b\}$	(a, b)	
$\{x a \leq x < b\}$	$[a, b)$	
$\{x a < x \leq b\}$	$(a, b]$	
$\{x a \leq x \leq b\}$	$[a, b]$	
$\{x x < a\}$	$(-\infty, a)$	
$\{x x \leq a\}$	$(-\infty, a]$	
$\{x x > a\}$	(a, ∞)	
$\{x x \geq a\}$	$[a, \infty)$	
\mathbb{R}	$(-\infty, \infty)$	

COMBINING SETS: UNIONS & INTERSECTIONS

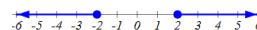
Suppose A and B are two sets

- The Intersection of A and B : $A \cap B = \{x | x \in A \text{ and } x \in B\}$
- The Union of A and B : $A \cup B = \{x | x \in A \text{ or } x \in B\}$
- Example:



UNION OF TWO INTERVALS

Suppose A and B are two intervals:



- The Union of A and B in set-builder notation:
 $A \cup B = \{x | x \leq -2 \text{ or } x \geq 2\}$
- The Union of A and B in interval notation:
 $(-\infty, -2] \cup [2, \infty)$