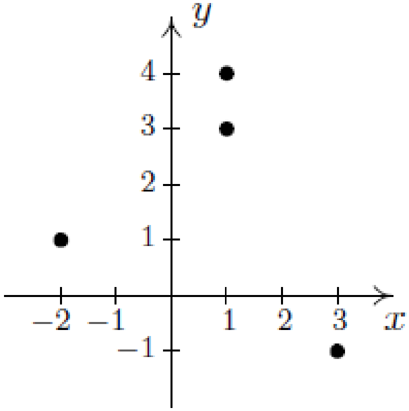
SECTION 1.3 & 1.4

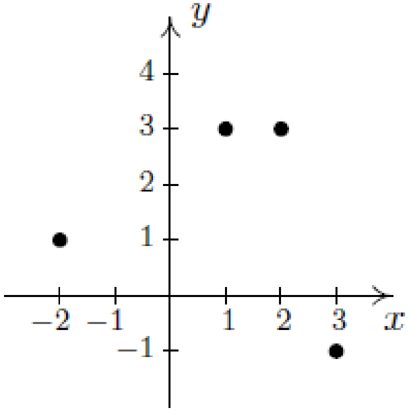
Introduction to Functions and Function Notation

Function Definition

- A relation in which each x-coordinate is matched with only one y-coordinate is said to describe y as a function of x
- Example: Which of the following relations describe y as a function of x?
- R_1= { (-2,1), (1,3), (1,4), (3,-1)}
- R_2= { (-2,1), (1,3), (2,3), (3,-1)}



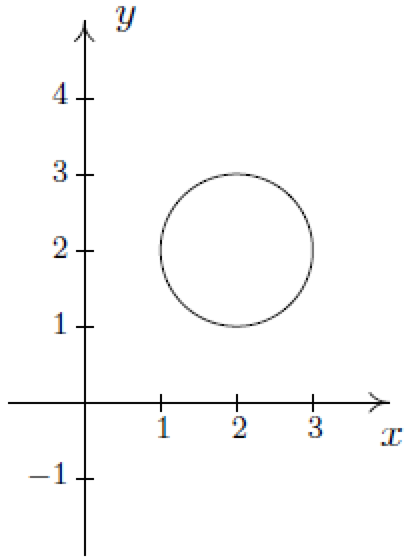
The graph of R_1

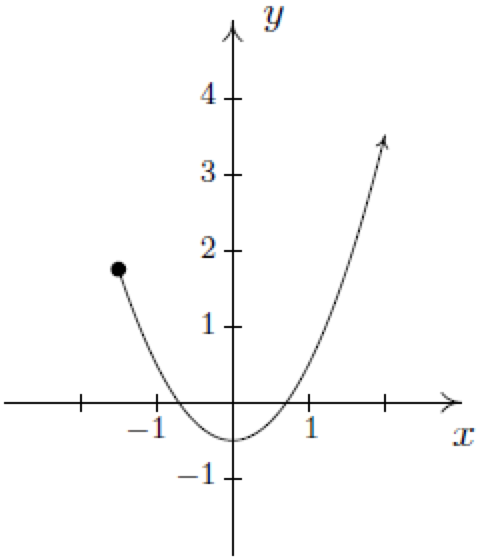


The graph of R_2

VERTICAL LINE TEST

- A set of points in the plane represents y as a function of x if and only if no two points lie on the same vertical line
- Example: Which of the following relations describes y as a function of x?

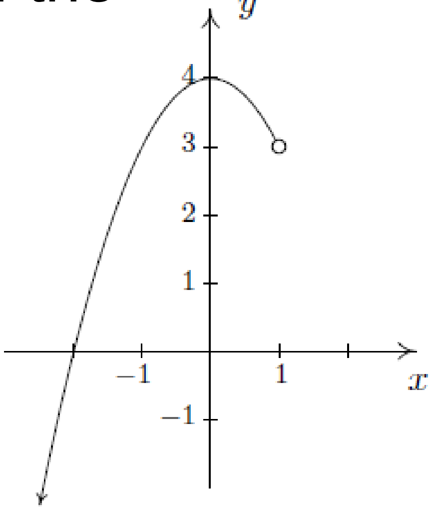




Domain and Range

- Suppose F is a relation which describes y as a function of x
- The set of the x-coordinates of the points in F is called the function **domain** of F
- The set of the y-coordinates of the points in F is called the function **range** of F
- Example: Find the domain and range of the following functions

F = { (-3, 2), (0, 1), (4, 2), (5, 2) }



FUNCTIONS AND RELATIONS

- All functions are relations
- But not all relations are functions
- Thus the equations which described the relations may or may not describe y as a function of x
- The algebraic representation of functions is possibly the most important way to view them so we need a process for determining whether or not an equation of a relation represents a function

Examples

- Determine which equations represent y as a function of x:
  - $x^3 + y^2 = 1$
  - $x^2 + y^3 = 1$
  - $x^2 y = 1 - 3y$
- For each of these equations, we solve for y and determine whether each choice of x will determine only one corresponding value of y

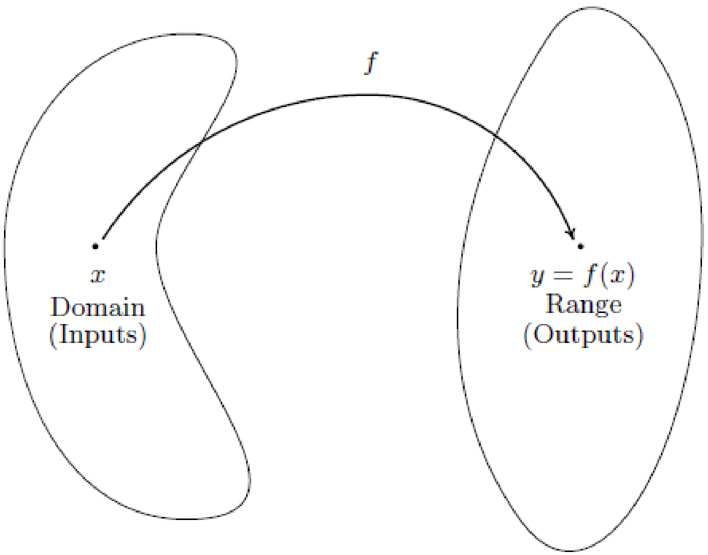
Examples

- For each of these equations, we solve for y and determine whether each choice of x will determine only one corresponding value of y

  - $x^3 + y^2 = 1 \rightarrow y^2 = 1 - x^3 \rightarrow y = \pm\sqrt{1 - x^3}$
  - $x^2 + y^3 = 1 \rightarrow y^3 = 1 - x^2 \rightarrow y = \sqrt[3]{1 - x^2}$
  - $x^2 y = 1 - 3y \rightarrow x^2 y + 3y = 1 \rightarrow y(x^2 + 3) = 1 \rightarrow y = 1/(x^2 + 3)$

Function as a Process

- The domain of a function is a set of inputs and
- The range is a set of outputs
- A function f is a process by which each input x is matched with only one output y
- y = f(x) is the output which results by applying the process f to the input x



Independent and Dependent Variables

- In the equation y=f(x)
  - The value of y is completely dependent on the choice of x.
  - x is called the independent variable or argument of f.
  - y is called the dependent variable.
  - When graphing the function, the independent variable goes on the horizontal axis and the dependent variable goes on the vertical axis.
- In the equation d=g(t)
  - What is the independent variable?
  - What is the dependent variable?

The Three Parts of a Function

A function consists of three parts:

1. An *input* represented by the *independent variable*, often the variable x. All possible valid input values are in a set called the *Domain* of the function.
2. An *output* represented by assigning the result of a function $f(x)$ to the *dependent variable*, often the variable y, as in $y = f(x)$. All possible valid output values are in a set called the *Range* of the function.
3. A *process* that *maps* each input value to one unique output value. The process is often represented by a
   - Description
   - Algebraic Expression
   - Table
   - Graph or Chart

Algebraic Formula of Function

- The process of a function f is usually described using an algebraic formula
- For example, suppose a function f takes a real number and performs the following two steps, in sequence
  1. multiply by 3
  2. add 4
- If 5 is our input, in step 1 we multiply by 3 to get (5)(3) = 15. In step 2, we add 4 to our result from step 1 which yields 15 + 4 = 19
- Using function notation, we would write $f(5) = 19$ to indicate that the result of applying the process f to the input 5 gives the output 19.
- In general, if we use x for the input, applying step 1 produces 3x. Following with step 2 produces $f(x) = 3x + 4$ as our final output.

Example

- For $f(x) = -x^2 + 3x + 4$, find and simplify

1. $f(-1)$, $f(0)$, $f(2)$
2. $f(2x)$, $2 f(x)$
3. $f(x + 2)$, $f(x) + 2$, $f(x) + f(2)$