Section 1.1

Part A: Sets of Numbers

Sets of Numbers

- A set is a well-defined collection of objects which are called the 'elements' of the set. The order of the elements do not matter.

- 'Well-defined' means that it is possible to determine if something belongs to the collection or not.

- When an element belongs to a set, we use the symbol '$\in$'

  If the element x belongs to set A, mathematically $x \in A$.

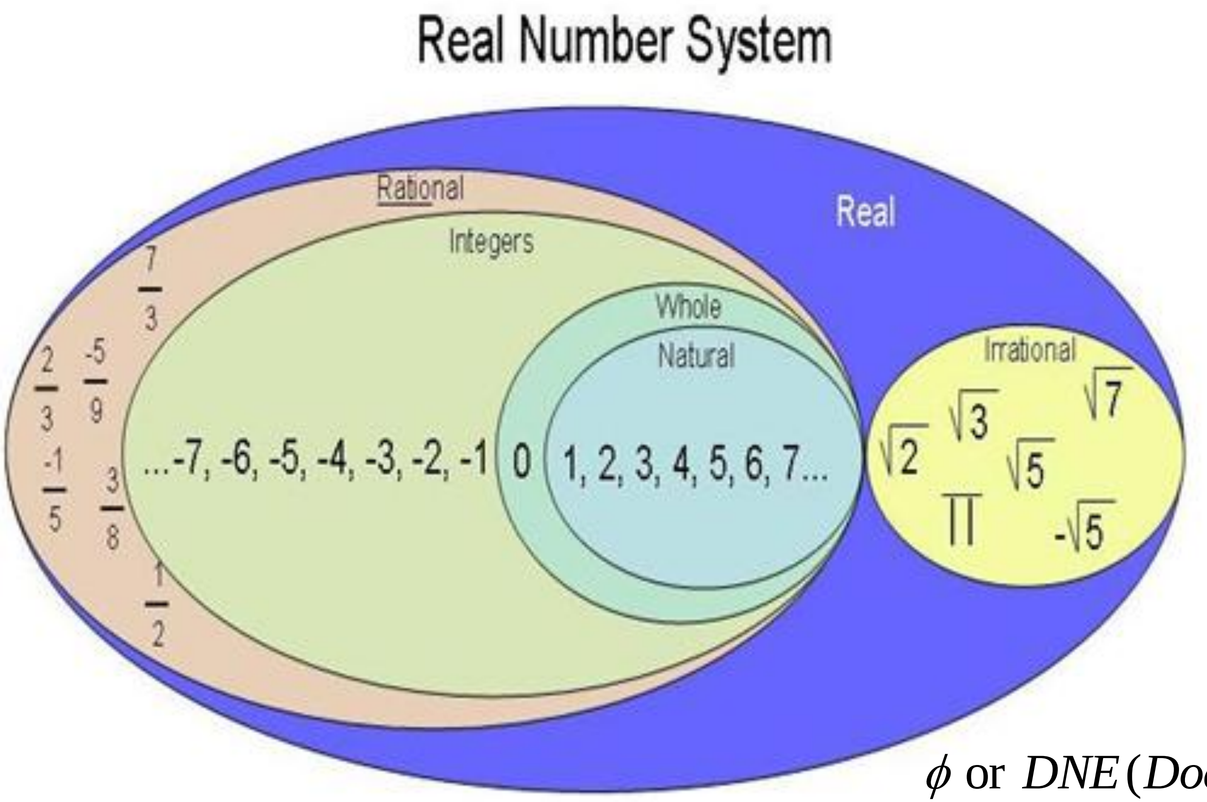
Ways to Describe Sets

- The Verbal Method: Use a sentence to define a set.
- The Roster Method: Begin with a left brace '{', list each element of the set only once and then end with a right brace '}'.
- The Set-Builder Method: A combination of the verbal and roster methods using a 'dummy variable' such as x.
- The Interval Notation Method: A method for writing an interval of numbers by using the endpoints of the interval.
- The Graphical Method: Graph interval of numbers on a number line.

Sets of Numbers

- The Empty Set: $\varnothing = \{\} = \{ x \mid x \neq x \}$. This is the set with no elements.
- The Natural Numbers: $\mathbb{N} = \{ 1, 2, 3, \ldots \}$
- The Whole Numbers: $\mathbb{W} = \{ 0, 1, 2, 3, \ldots \}$
- The Integers: $\mathbb{Z} = \{ \ldots , -3, -2, -1, 0, 1, 2, 3, \ldots\}$
- The Rational Numbers: $\mathbb{Q} = \{$ m/n where m, n are integers and $n \neq 0$ $\}$
- The Irrational Numbers: $\mathbb{P} = \{$ all numbers that cannot be written as a rational number $\}$
- The Real Numbers: $\mathbb{R} = \{$ all numbers on the number line; both rational and irrational $\}$.

Real Number System



ϕ or $DNE\,(Does\ Not\ Exist)$

GRAPHICAL METHOD

In graphing a set of numbers on a number line, there are 5 things to draw:

- Draw a line segment in bold (or in color) to show an interval of numbers.
- Put an open circle at the end of a line segment to exclude the end point from the interval.
- Put a filled-in circle at the end of a line segment to include the end point of the interval.
- Put a left arrow at the end of a line segment to show that the segment continues to negative infinity.
- Put a right arrow at the end of a line segment to show that the segment continues to infinity.

Set-Builder Notation

In set-builder notation:

- Start with "**{ *x* |**" that means the set of ***x*** where conditions on ***x*** are to follow after the |. Note: ***x*** is a dummy variable and any dummy variable can be used.
- Conditions are expressed as equalities or inequalities of *x*, such as:
  - $x = 2$
  - $x < 2$, $x \leq 2$, $x > 2$, $x \geq 2$
  - $-2 < x \leq 3$
- Do not write inequalities using the ∞ symbol.
- Join equalities or inequalities with the word "**or**".
- Finish the notation with "**}**"

Interval Notation

In interval notation, there are 5 symbols to know:

- Open parentheses excludes end points ( )
- Closed brackets include end points [ ]
- Infinity: ∞
- Negative Infinity: $-\infty$
- Union Symbol: $\cup$

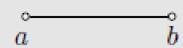
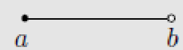
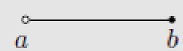
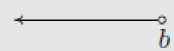
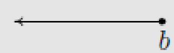
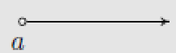
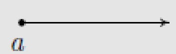
INTERVAL NOTATION TIPS

- All numbers within the interval notation are in ascending numeric order from left to right.
- If the interval goes on forever to the left, start the interval notation with $(-\infty$
- Similarly, if the interval goes on forever to the right, end the interval notation with $\infty)$
- Whenever a graph consists of multiple line segments, write out each interval from left to right. Put the union $\cup$ symbol between each interval to join them together.

Interval Notation Examples

Interval Notation

Let a and b be real numbers with $a < b$.

Set of Real Numbers	Interval Notation	Region on the Real Number Line
$\{x \mid a < x < b\}$	(a, b)	
$\{x \mid a \leq x < b\}$	$[a, b)$	
$\{x \mid a < x \leq b\}$	$(a, b]$	
$\{x \mid a \leq x \leq b\}$	$[a, b]$	
$\{x \mid x < b\}$	$(-\infty, b)$	
$\{x \mid x \leq b\}$	$(-\infty, b]$	
$\{x \mid x > a\}$	(a, ∞)	
$\{x \mid x \geq a\}$	$[a, \infty)$	
$\mathbb{R}$	$(-\infty, \infty)$	

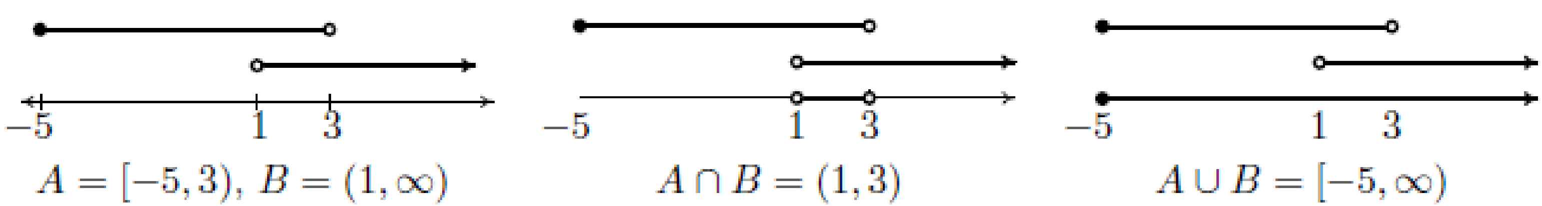
Combining Sets: Unions & Intersections

Suppose ***A*** and ***B*** are two sets

- The Intersection of ***A*** and ***B***: $A \bigcap B = \{x \mid x \in A \text{ and } x \in B\}$

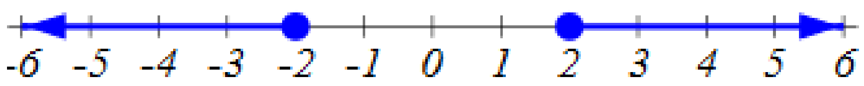
- The Union of ***A*** and ***B*** : $A \bigcup B = \{x \mid x \in A \text{ or } x \in B\}$

- Example:



Union of Two Intervals

Suppose ***A*** and ***B*** are two intervals:



- The Union of ***A*** and ***B*** in set-builder notation:

$$A \cup B = \{x \mid x \leq -2 \text{ or } x \geq 2\}$$

- The Union of ***A*** and ***B*** in interval notation:

$$(-\infty, -2] \cup [2, \infty)$$