Section 1.5

Function Arithmetic

Function Arithmetic (Addition and Subtraction)

Suppose f and g are functions and x is an element common to the domains of f and g.

- The sum of f and g, denoted f + g, is the function defined by the formula:

$$(f + g)(x) = f(x) + g(x)$$

- The difference of f and g, denoted f - g, is the function defined by the formula:

$$(f - g)(x) = f(x) - g(x)$$

- If F is the domain of function f and G is the domain of function g, then the domain of either (f + g)(x) or (f - g)(x) is the intersection of the two domains:

$$F \cap G$$

Function Arithmetic (Multiplication)

Suppose f and g are functions and x is an element common to the domains of f and g.

- The product of f and g, denoted fg, is the function defined by the formula:

$$(fg)(x) = f(x)g(x)$$

- If F is the domain of function f and G is the domain of function g, then the domain of (fg)(x) is the intersection of the two domains:

$$F \cap G$$

Function Arithmetic (Division)

Suppose f and g are functions and x is an element common to the domains of f and g.

- The quotient of f and g, denoted f/g, is the function defined by the formula:

  $(f/g)(x)=f(x)/g(x)$, provided $g(x) \neq 0$

- If F is the domain of function f and G is the domain of function g, then the domain of (f/g)(x) is the intersection of the two domains and where $g(x) \neq 0$:

$$\{x \in F \cap G \mid g(x) \neq 0\}$$

Examples

- Let $f(x) = 6x^2 - 2x$ and $g(x) = 3 - 1/x$
- Evaluate

1. $(f + g)(-1) = f(-1) + g(-1) \rightarrow$

   $$[6(-1)^2 - 2(-1)] + [(3 - 1/(-1)] \rightarrow 8 + 4 \rightarrow 12$$

2. $(f\ g)(2) = f(2)g(2) \rightarrow$

   $$[6(2)^2 - 2(2)][(3 - 1/(2)] \rightarrow (20)(5/2) \rightarrow 50$$

EXAMPLES

- Let f(x) = 6x^2 - 2x and g(x) = 3 - 1/x
- Find and simplify expressions for
  1. (f - g)(x) = f(x) – g(x) $\rightarrow$

$$[6x^2-2x]-[3-1/x]\rightarrow 6x^2-2x-3+1/x\rightarrow \frac{6x^3-2x^2-3x+1}{x}$$

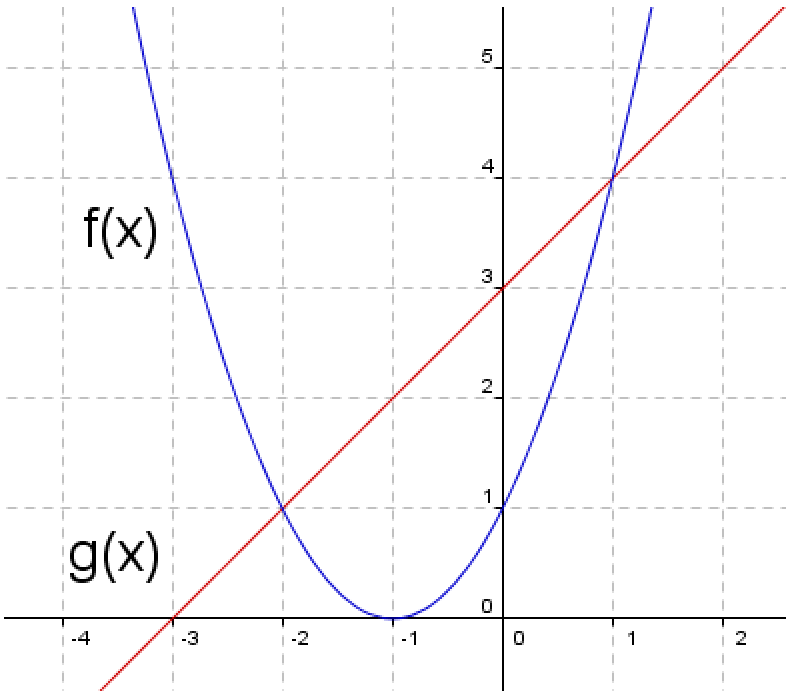
  2. (f/g)(x) = f(x)/g(x) $\rightarrow$

$$\frac{6x^2-2x}{3-1/x}\rightarrow \frac{6x^2-2x}{\frac{3x-1}{x}}\rightarrow \frac{6x^2-2x}{1}\frac{x}{3x-1}\rightarrow \frac{6x^3-2x^2}{3x-1}\rightarrow$$

$$\frac{2x^2(3x-1)}{3x-1}\rightarrow 2x^2$$

Examples

- Let f and g be functions defined by the graph



- Find the indicated value
  - $(f + g)(0) = f(0) + g(0) \rightarrow$
  - $(f\,g)(-2) = f(-2)g(-2) \rightarrow$

Examples

- Let f and g be functions defined by the table

x	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**
$f(x)$	4	2	0	1	3	4	-1
$g(x)$	-2	0	-4	0	-3	1	2

- Find the indicated value
  - $(f + g)(-3) = f(-3) + g(-3) \rightarrow$
  - $(f\,g)(3) = f(3)g(3) \rightarrow$

EXAMPLES

- Let f be a function defined by the roster notation

  $$f = \{(-3,4),(-2,2),(-1,0),(0,1),(1,3),(2,4),(3,-1)\}$$

- and let g be a function defined

  $$g = \{(-3,-2),(-2,0),(-1,-4),(0,0),(1,-3),(2,1),(3,2)\}$$

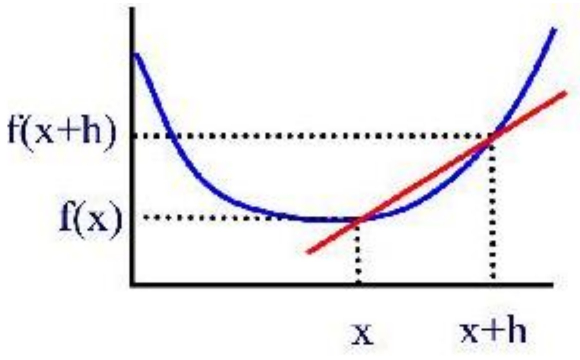
- Find the indicated value
  - $(f+g)(-3) = f(-3) + g(-3) \rightarrow$
  - $(fg)(3) = f(3)g(3) \rightarrow$

Difference Quotient

- Given a function, f, difference quotient of f is the expression:

$$\frac{f(x+h)-f(x)}{h}$$

- Geometric meaning: the difference quotient is the slope of the secant line.



Simplifying the Difference Quotient

- For a given function f(x), simplifying the difference quotient means rewriting it in a form where the ‘h’ in the definition of the difference quotient cancels from the denominator.

$$\frac{f(x+h)-f(x)}{h} \rightarrow ?$$

Difference Quotient Example

- Find and simplify the difference quotient for the function: $f(x) = x^2 - x - 2$

$$\frac{f(x+h)-f(x)}{h} \rightarrow \frac{[(x+h)^2-(x+h)-2]-[x^2-x-2]}{h} \rightarrow$$

$$\frac{x^2+2hx+h^2-x-h-2-x^2+x+2}{h} \rightarrow \frac{2xh+h^2-h}{h} \rightarrow$$

$$\frac{h(2x+h-1)}{h} \rightarrow 2x+h-1$$