Section 1.7

Transformations: Shifts, Reflections, and Scalings

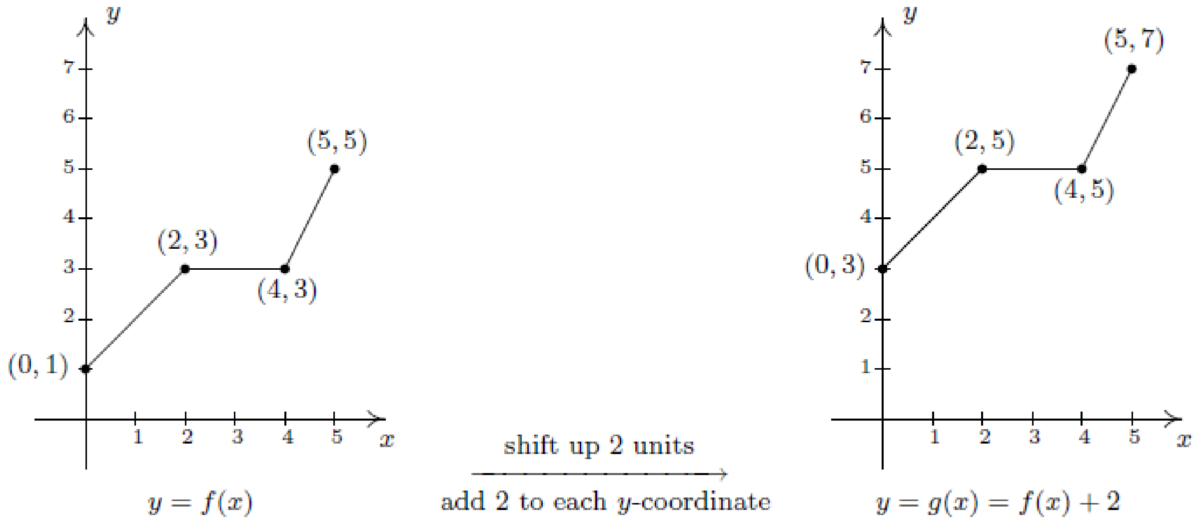
VERTICAL SHIFTS

- Suppose f is a function and k is a positive number
- To graph $y = f(x) + k$, shift the graph of $y = f(x)$ up k units by adding k to the y-coordinates of the points on the graph of f
- To graph $y = f(x) - k$, shift the graph of $y = f(x)$ down k units by subtracting k from the y-coordinates of the points on the graph of f

Shift Up

- g(x)= f(x) + 2

x	$f(x)$	$g(x) = f(x) + 2$	$(x, g(x))$
0	$f(0) = 1$	$g(0) = 3$	(0, 1)
2	$f(2) = 3$	$g(2) = 5$	(2, 3)
4	$f(4) = 3$	$g(4) = 5$	(4, 3)
5	$f(5) = 5$	$g(5) = 7$	(5, 5)



Horizontal Shifts

- Suppose f is a function and h is a positive number.

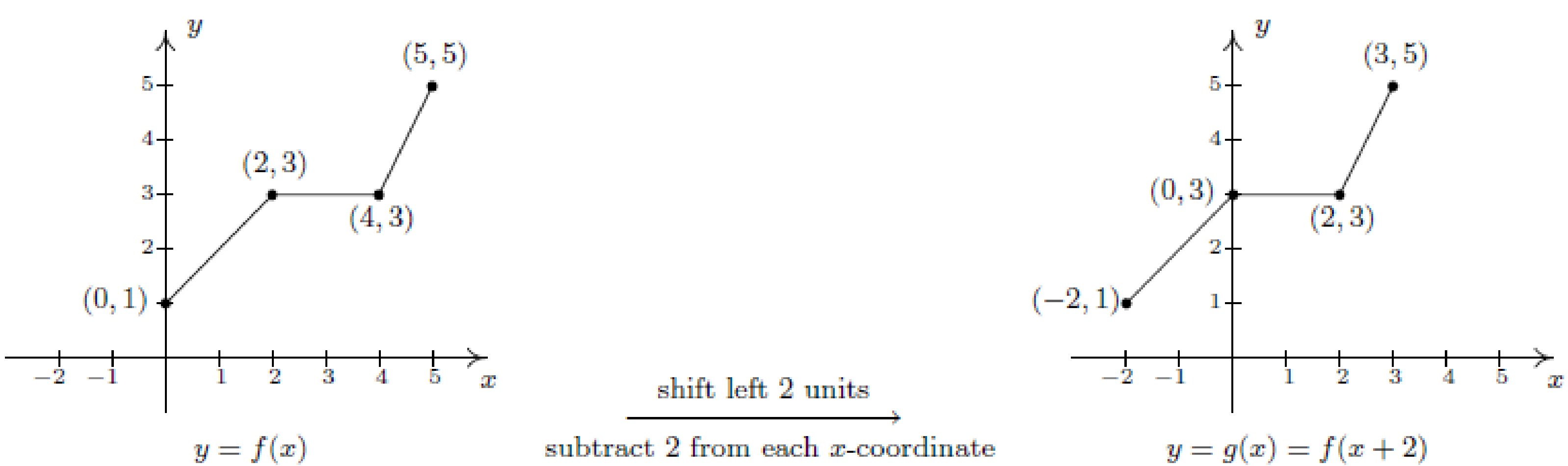
- To graph $y = f(x + h)$, shift the graph of $y = f(x)$ left h units by subtracting h from the x-coordinates of the points on the graph of f.

- To graph $y = f(x - h)$, shift the graph of $y = f(x)$ right h units by adding h to the x-coordinates of the points on the graph of f.

Shift Left

- g(x)= f(x+2)

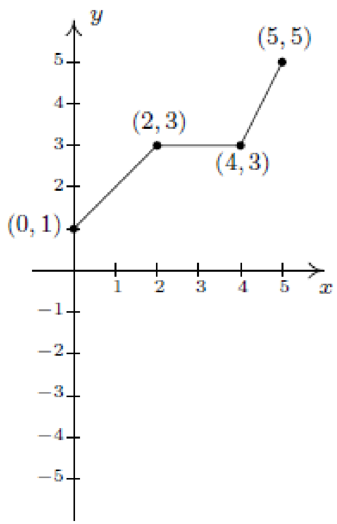
Left Graph		Right Graph				
x	***f*** **(*x*)**	***f*** **(*x*+2)**	***x*+2**	***x***	***g*(*x*) = *f*** **(*x*+2)**	**(*x, g*(*x*))**
0	*f* (0) = 1	*f* (0) = 1	0	-2	*g*(-2) = 1	(-2, 1)
2	*f* (2) = 3	*f* (2) = 3	2	0	*g*(0) = 3	(0, 3)
4	*f* (4) = 3	*f* (4) = 3	4	2	*g*(2) = 3	(2, 3)
5	*f* (5) = 5	*f* (5) = 5	5	3	*g*(3) = 5	(3, 5)



Reflections

- Suppose f is a function.
- To graph y = -f(x), reflect the graph of y = f(x) across the x-axis by multiplying the y-coordinates of the points on the graph of f by -1.
- To graph y = f(-x), reflect the graph of y = f(x) across the y-axis by multiplying the x-coordinates of the points on the graph of f by -1.

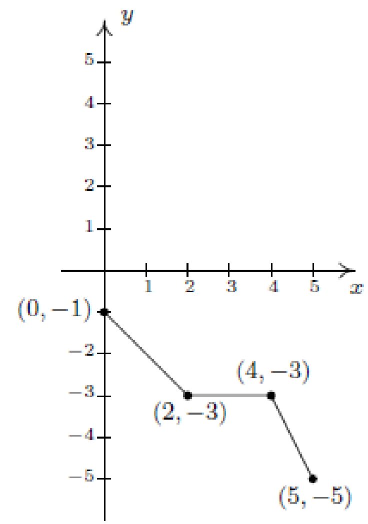
Example



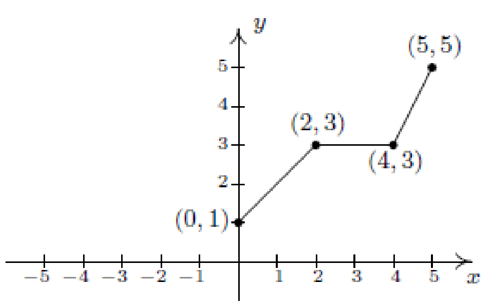
$y = f(x)$

reflect across x-axis

multiply each y-coordinate by -1



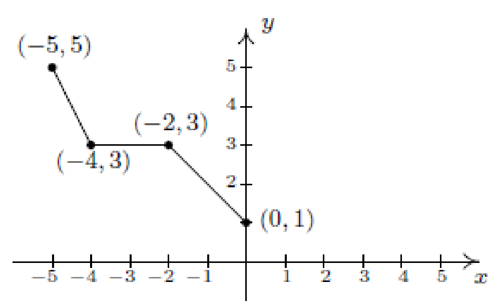
$y = -f(x)$



$y = f(x)$

reflect across y-axis

multiply each x-coordinate by -1



$y = f(-x)$

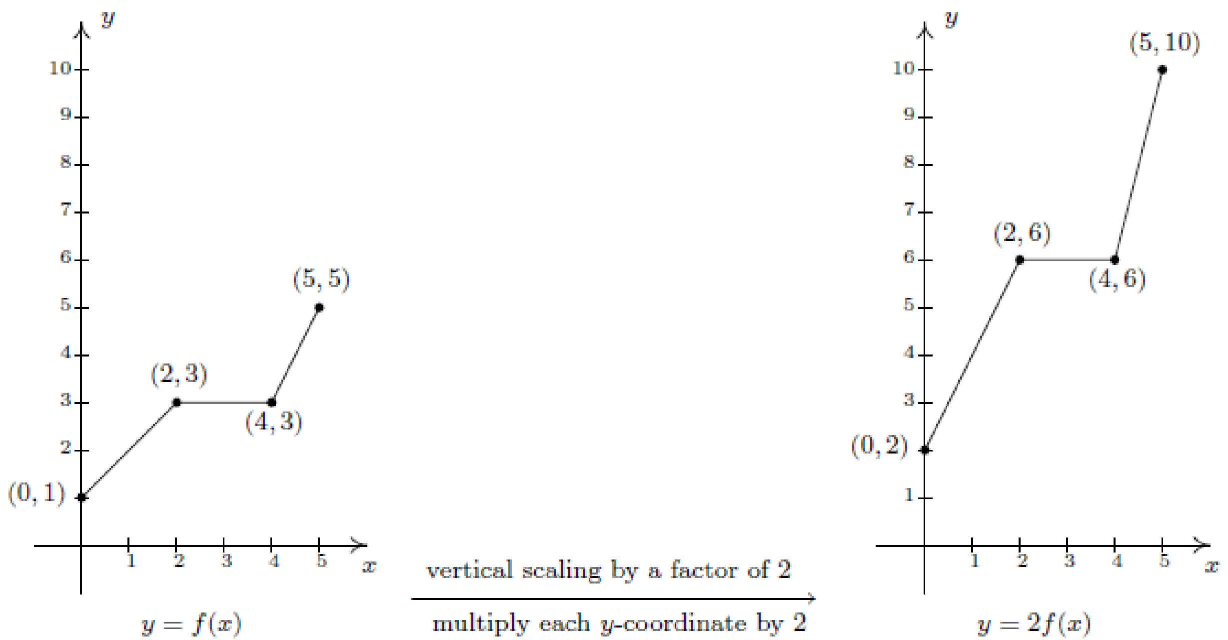
Vertical Scaling

- Suppose f is a function and $a > 0$.
- To graph $y = a\,f(x)$, multiply all of the y-coordinates of the points on the graph of f by a.
- We say the graph of f has been vertically scaled by a factor of a.
  - If $a > 1$, we say the graph of f has undergone a vertical stretch (expansion, dilation) by a factor of a.
  - If $0 < a < 1$, we say the graph of f has undergone a vertical shrink (compression, contraction) by a factor of $1/a$.

Vertical Scaling

Graph g(x) = 2f(x)

x	$f(x)$	$g(x) = 2f(x)$	$(x, g(x))$
0	$f(0) = 1$	$g(0) = 2$	(0, 2)
2	$f(2) = 3$	$g(2) = 6$	(2, 6)
4	$f(4) = 3$	$g(4) = 6$	(4, 6)
5	$f(5) = 5$	$g(5) = 10$	(5, 10)



Horizontal Scaling

- Suppose f is a function and $b > 0$.

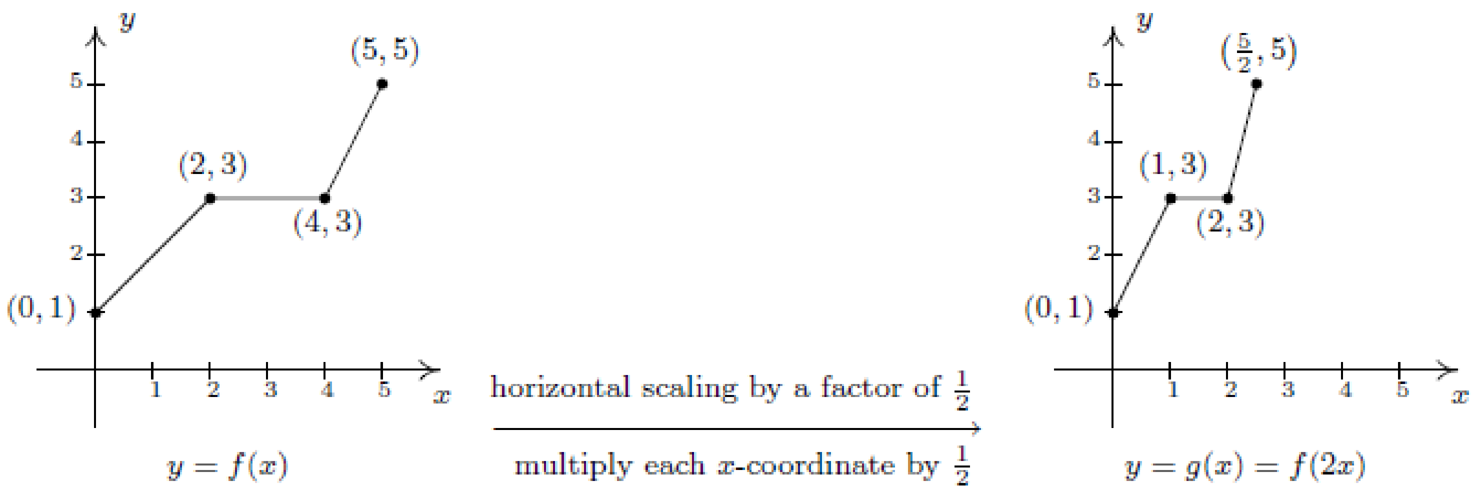
- To graph $y = f(bx)$, divide all of the x-coordinates of the points on the graph of f by b.

- We say the graph of f has been horizontally scaled by a factor of $1/b$.
  - If $0 < b < 1$, we say the graph of f has undergone a horizontal stretch (expansion, dilation) by a factor of $1/b$.
  - If $b > 1$, we say the graph of f has undergone a vertical shrink (compression, contraction) by a factor of b.

Horizontal Scaling

- Graph *g(x)* = *f(2x)*

Left Graph		Right Graph				
x	**$f(x)$**	**$f(2x)$**	**$2x$**	**x**	**$g(x) = f(2x)$**	**$(x, g(x))$**
0	$f(0) = 1$	$f(0) = 1$	0	0	$g(0) = 1$	(0, 1)
2	$f(2) = 3$	$f(2) = 3$	2	1	$g(1) = 3$	(1, 3)
4	$f(4) = 3$	$f(4) = 3$	4	2	$g(2) = 3$	(2, 3)
5	$f(5) = 5$	$f(5) = 5$	5	2.5	$g(2.5) = 5$	(2.5, 5)



ALGORITHM FOR TRANSFORMATIONS

Suppose f is a function. To graph $g(x) = Af(Bx + H) + K$

1. Subtract H from each of the x-coordinates of the points on the graph of f. This results in a horizontal shift to the
   - left if H > 0 (positive H)
   - right if H< 0 (negative H)
2. Divide the x-coordinates of the points on the graph obtained in Step 1 by B. This results in a horizontal scaling, but may also include a reflection about the y-axis if B < 0 (negative B)
3. Multiply the y-coordinates of the points on the graph obtained in Step 2 by A. This results in a vertical scaling, but may also include a reflection about the x-axis if A < 0 (negative A)
4. Add K to each of the y-coordinates of the points on the graph obtained in Step 3. This results in a vertical shift
   - up if K > 0 (positive K)
   - down if K< 0 (negative K)