

SECTION 1.5

Function Arithmetic

FUNCTION ARITHMETIC (ADDITION AND SUBTRACTION)

Suppose f and g are functions and x is an element common to the domains of f and g .

- The sum of f and g , denoted $f + g$, is the function defined by the formula:

$$(f + g)(x) = f(x) + g(x)$$

- The difference of f and g , denoted $f - g$, is the function defined by the formula:

$$(f - g)(x) = f(x) - g(x)$$

- If F is the domain of function f and G is the domain of function g , then the domain of either $(f + g)(x)$ or $(f - g)(x)$ is the intersection of the two domains:

$$F \cap G$$

FUNCTION ARITHMETIC (MULTIPLICATION)

Suppose f and g are functions and x is an element common to the domains of f and g .

- The product of f and g , denoted fg , is the function defined by the formula:

$$(fg)(x) = f(x)g(x)$$

- If F is the domain of function f and G is the domain of function g , then the domain of $(fg)(x)$ is the intersection of the two domains:

$$F \cap G$$

FUNCTION ARITHMETIC (DIVISION)

Suppose f and g are functions and x is an element common to the domains of f and g .

- The quotient of f and g , denoted f/g , is the function defined by the formula:

$$(f/g)(x) = f(x)/g(x), \text{ provided } g(x) \neq 0$$

- If F is the domain of function f and G is the domain of function g , then the domain of $(f/g)(x)$ is the intersection of the two domains and where $g(x) \neq 0$:

$$\{x \in F \cap G \mid g(x) \neq 0\}$$

EXAMPLES

- Let $f(x) = 6x^2 - 2x$ and $g(x) = 3 - 1/x$

- Evaluate

$$1. (f + g)(-1) = f(-1) + g(-1) \rightarrow [6(-1)^2 - 2(-1)] + [(3 - 1/(-1))] \rightarrow 8 + 4 \rightarrow 12$$

$$2. (fg)(2) = f(2)g(2) \rightarrow [6(2)^2 - 2(2)][(3 - 1/(2))] \rightarrow (20)(5/2) \rightarrow 50$$

EXAMPLES

- Let $f(x) = 6x^2 - 2x$ and $g(x) = 3 - 1/x$

- Find and simplify expressions for

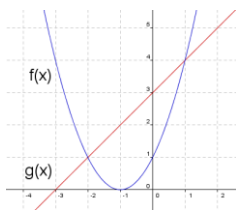
$$1. (f - g)(x) = f(x) - g(x) \rightarrow [6x^2 - 2x] - [3 - 1/x] \rightarrow 6x^2 - 2x - 3 + 1/x \rightarrow \frac{6x^3 - 2x^2 - 3x + 1}{x}$$

$$2. (f/g)(x) = f(x)/g(x) \rightarrow \frac{6x^2 - 2x}{3 - 1/x} \rightarrow \frac{6x^2 - 2x}{\frac{3x - 1}{x}} \rightarrow \frac{6x^2 - 2x}{1} \cdot \frac{x}{3x - 1} \rightarrow \frac{6x^3 - 2x^2}{3x - 1}$$

$$\frac{2x^2(3x - 1)}{3x - 1} \rightarrow 2x^2$$

EXAMPLES

- Let f and g be functions defined by the graph



- Find the indicated value
 - $(f + g)(0) = f(0) + g(0) \rightarrow$
 - $(fg)(-2) = f(-2)g(-2) \rightarrow$

EXAMPLES

- Let f and g be functions defined by the table

x	-3	-2	-1	0	1	2	3
$f(x)$	4	2	0	1	3	4	-1
$g(x)$	-2	0	-4	0	-3	1	2

- Find the indicated value
 - $(f + g)(-3) = f(-3) + g(-3) \rightarrow$
 - $(fg)(3) = f(3)g(3) \rightarrow$

EXAMPLES

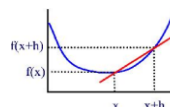
- Let f be a function defined by the roster notation
 $f = \{(-3, 4), (-2, 2), (-1, 0), (0, 1), (1, 3), (2, 4), (3, -1)\}$
- and let g be a function defined
 $g = \{(-3, -2), (-2, 0), (-1, -4), (0, 0), (1, -3), (2, 1), (3, 2)\}$
- Find the indicated value
 - $(f + g)(-3) = f(-3) + g(-3) \rightarrow$
 - $(fg)(3) = f(3)g(3) \rightarrow$

DIFFERENCE QUOTIENT

- Given a function, f , difference quotient of f is the expression:

$$\frac{f(x+h) - f(x)}{h}$$

- Geometric meaning: the difference quotient is the slope of the secant line.



SIMPLIFYING THE DIFFERENCE QUOTIENT

- For a given function $f(x)$, simplifying the difference quotient means rewriting it in a form where the 'h' in the definition of the difference quotient cancels from the denominator.

$$\frac{f(x+h) - f(x)}{h} \rightarrow ?$$

DIFFERENCE QUOTIENT EXAMPLE

- Find and simplify the difference quotient for the function:
 $f(x) = x^2 - x - 2$

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &\rightarrow \frac{[(x+h)^2 - (x+h) - 2] - [x^2 - x - 2]}{h} \rightarrow \\ \frac{x^2 + 2hx + h^2 - x - h - 2 - x^2 + x + 2}{h} &\rightarrow \frac{2xh + h^2 - h}{h} \rightarrow \\ \frac{h(2x + h - 1)}{h} &\rightarrow 2x + h - 1 \end{aligned}$$